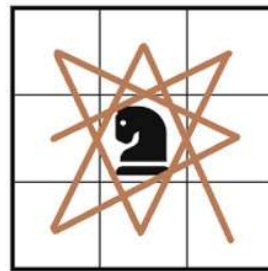
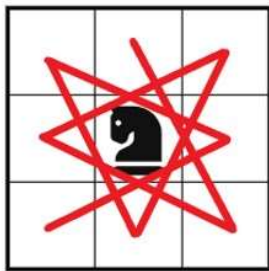
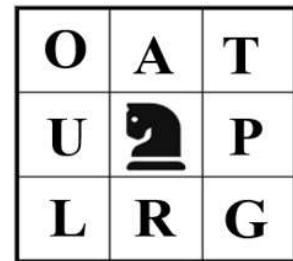
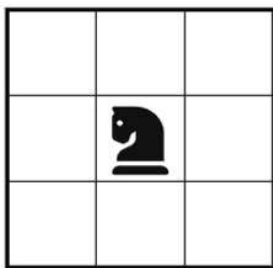


The Knight's Tour: Where the Autonomous Template Provides the Sequence—Reversing the Learning Direction from Procedural to Declarative

The complete explanation of all functional perceptual processes within sequential learning



Caught In A Line

The explanatory model of all motoric movement actions

N.J. Mol
March, 2025

Contact: kwillinq@gmail.com
<https://www.researchgate.net/profile/Nj-Mol/research>
<https://www.explanatorymodel.nl/>

Abstract

The comprehensive explanation of all functional perceptual processes within sequential learning has recently been uncovered. It demonstrates that an autonomous spatiotemporal template of action trajectory shapes emerges *between* (!) the sequence items through the execution of those consecutive items themselves. This discovery provides a definitive and conclusive answer to the essential research questions in this field regarding the what and how within sequential learning.

The “what (must be reached)”, within the symbolic-numerical representation, specifies the script items that must be attained, while the “how (must it be reached)”, within the spatiotemporal representation, defines how the empty dimension *between* (!) these script items must be bridged. This process has been extensively analyzed in tasks such as keypad-based code entry, piano playing, and supermarket navigation, where the emerging template increasingly governs task execution—or in other words, where the how evolves through the what.

However, the Knight's Tour puzzle reveals an inverse yet equally autonomous mechanism of sequential learning. In this case, the action trajectory template is predefined by the movement constraints of the chess knight, rather than emerging dynamically through execution. Instead of constructing a trajectory through repetition, the solver must extract the correct symbolic sequence from an already existing cognitive template. This reversal demonstrates that action trajectory templates do not merely emerge from symbolic encoding but can autonomously dictate execution in problem-solving contexts.

By contrasting the Knight's Tour puzzle with other sequential learning tasks, this study establishes that autonomous action templates can structure task completion even in the absence of explicit symbolic instruction. This challenges traditional models of motor learning by demonstrating that execution is not solely dependent on symbolic representation but can also be guided by an inherent spatiotemporal structure, with profound implications for cognitive science, artificial intelligence, and neurocognitive modeling.

Keywords: sequential learning, autonomous action templates, motor cognition, Knight's Tour puzzle, symbolic-to-motor transition, motor-to-symbolic discovery, cognitive problem-solving.

Introduction

Motor sequence learning is a fundamental cognitive process essential for skill acquisition and has been extensively studied within cognitive and neuroscientific research. Repeated execution of a movement sequence has been shown to improve performance efficiency, suggesting that the brain encodes previously executed sequences as a form of motor memory. However, despite decades of research, a critical gap remains in understanding how motor and symbolic representations interact. Existing theories predominantly maintain a strict separation between declarative memory (explicit knowledge, such as remembering a numerical code) and procedural memory (implicit knowledge, such as performing a motor sequence without conscious awareness). This perspective implies that motor learning involves a gradual shift from declarative encoding to procedural automation through repetition.

However, recent insights challenge this dichotomy by proposing that motor and symbolic knowledge are not separate systems but cognitive variants of the same underlying principle. The explanatory model of the motoric movement action (2016) introduced a novel framework in which motor sequences are not merely stored as isolated actions or rigid patterns but as cognitive templates of action trajectory shapes. These templates encode both the temporal and spatial structure of an action, allowing for flexible and autonomous execution. For example, when entering a numerical code on a keypad,

early performance depends on the explicit recognition of individual numbers and their sequence. Over time, however, an implicit spatiotemporal template of action trajectory shapes emerges, initially derived from declarative knowledge but eventually capable of autonomously guiding task execution—even in the absence of explicit perception of the numerical code.

This perspective provides a conclusive explanation of the fundamental research questions in sequential learning, addressing both the what and how of action execution. The “what”, within the symbolic-numerical representation, specifies which script items must be reached, while the “how”, within the spatiotemporal representation, defines the empty dimension between these script items. Previous research has extensively demonstrated that these representations can function autonomously but are most commonly employed in a hybrid form, dynamically integrating symbolic and motor processes.

While this explanatory model has been validated across various sequential tasks—such as keypad-based code entry, piano playing, and navigation in supermarkets—the Knight's Tour puzzle reveals an inverse yet equally autonomous mechanism. Unlike traditional motor learning paradigms where execution refines an emerging action trajectory template, in the Knight's Tour puzzle, the spatiotemporal template is pre-existing, dictated by the movement constraints of the chess knight. Rather than forming a trajectory through execution, the solver must extract the correct symbolic sequence from an already defined motor structure. This reversal challenges the prevailing notion that action trajectory templates arise solely from symbolic encoding and demonstrates that such templates can autonomously dictate execution, even in problem-solving contexts.

By examining the Knight's Tour puzzle alongside other sequential learning tasks, this study highlights the bidirectional nature of motor cognition. It establishes that autonomous action templates can structure execution even in the absence of explicit symbolic instruction, challenging traditional models of motor learning. This finding has significant implications for our understanding of cognitive representation, motor learning, artificial intelligence, and neurocognitive processing. If further empirically validated, this model has the potential to fundamentally transform research on skill acquisition, neural network modeling, and cognitive flexibility in problem-solving.

1. The explanatory model of the motoric movement action

In 2016, an explanatory model for motor execution was developed that provides a framework for identifying all functional perceptual processes within any goal-directed motor action¹. This model offers a universal explanation, demonstrating that performing any action necessarily requires the simultaneous perception of three autonomous foci. One of these foci remains directed at the movement of the environmental object, which universally represents a catching action. The two other foci are engaged in perceiving the movement within the egocentric execution of the action, which can universally be described as a throwing action.

Within the present scientific study, the focus is on this egocentric throwing action, and the model demonstrates that a perceptual representation of a latent action trajectory shape is always formed in advance. This representation encodes how future, *consecutive* (!) positions of the end-effectors, i.e., the fingertips, will successfully align within the action trajectory shape. The critical insight is that the interaction between the movement of the fingertips within the action trajectory shape:

1. Must be viewed as a single, undivided phenomenon,
2. Provides the ultimate explanation of the perception-action coupling, and
3. Integrates all previously discovered scientific phenomena.

¹ <https://www.explanatorymodel.nl/>.

The movement of the end-effectors within the action trajectory shape thus constitutes a single, indivisible phenomenon that encapsulates multiple dimensions. This article establishes that the shape of a single action trajectory underlies the broader template c.q. the pattern of action trajectory shapes that emerges within sequential actions.

2. The action trajectory shape and templates of action trajectory shapes in sequential actions

This paragraph establishes the foundation for understanding the template of action trajectory shapes. It describes the transition from the shape of one single action trajectory to the more abstract templates of multiple action trajectory shapes that emerge in recent scientific research. This process illustrates that motor actions do not merely consist of isolated movements but are cognitively represented within mental structures that remain fundamentally consistent despite individual variability.

a. The shape of a single action trajectory shape – Inserting a car key into the ignition

When a driver moves a car key toward the ignition slot², the key does not follow a straight linear path. Instead, the movement is executed in a geometrically curved trajectory shape, determined by the ego-centric throwing action of the key and the spatial constraints of the steering column. Although the precise curvature of the action trajectory shape varies between different car models and steering column configurations, universal characteristics can be observed in the shape of this movement: 1. A systematic bending point in the action trajectory shape, occurring at the widest part of the steering wheel, 2. A consistent trajectory length, characterized by a *tau*-value that determines the time-to-contact (ttc), and 3. Adaptability to variations in vehicle configuration, where a new car requires an initial visual scan, whereas in a familiar car, the key is inserted blindly into the ignition.



In an unfamiliar vehicle, this process must first be calibrated, which often provokes the visual location of the ignition slot. However, with repeated execution, an implicit perceptual image of the action trajectory shape develops, which is internalized as a cognitive template after just a few repetitions. This results in an autonomous representation capable of guiding the motor execution without any visual control.

This example illustrates that action trajectory shapes are not merely motor trajectories but perceptual representations of the movement of the end-effector, in this case, the car key. The cognitive representation of a latent action trajectory shape serves as the autonomous primary focus, guiding the key's motion, while motor execution functions as the autonomous secondary focus, merely executing this representation. The autonomy of these two foci explains why the key consistently follows a similarly shaped curvature, yet is never capable of exactly replicating the same trajectory (Bernstein).

b. Templates of action trajectory shapes in sequential actions – Unlocking a lock with a numerical code

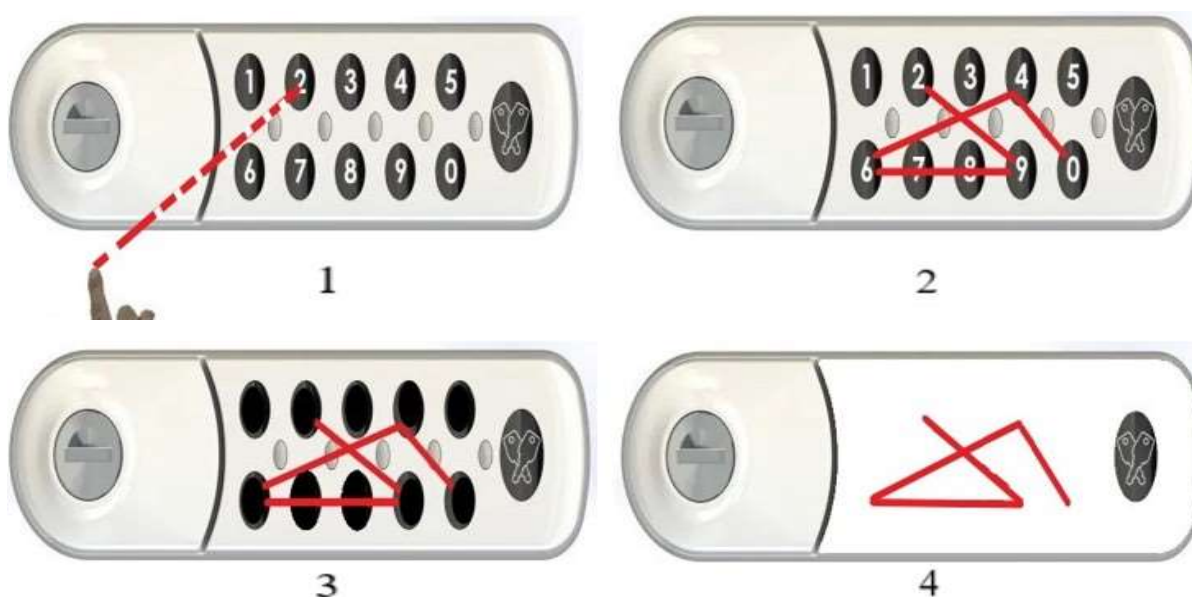
Building upon the single action trajectory shape described in the previous section, the analysis now extends to sequential motor actions, in which a script of multiple action trajectories must be executed.

² General lock opening with a key is extensively explained: <https://www.explanatorymodel.nl/common-daily-actions/opening-a-lock-with-a-key>.

This constitutes a crucial step toward the subsequent section, where even more abstract sequential actions will be examined.

Some locks operate via a keypad mechanism, where entering a specific access code, typically consisting of four or five digits, unlocks the lock. Suppose a user must enter the code 29640 for the first time. The initial action consists of consciously locating the key labeled "2" and directing the index finger toward it. Unlike the car key example, where the movement followed a curved trajectory due to spatial constraints, this action follows a straighter action trajectory shape since there is no steering column where you need to navigate around. However, the underlying perceptual processes remain the same: the action trajectory shape remains a cognitive representation that directs the movement. After pressing "2," the index finger then moves toward "9," again following a relatively straight trajectory. This process repeats for each subsequent digit (6 → 4 → 0), with each action trajectory initially being planned and executed separately.

Even after the first successful execution, an integrated mental representation of the overall action pattern of action trajectory shapes begins to form. On subsequent attempts, elements of this template of action trajectory shapes are activated, allowing the user to perform the task more efficiently. Instead of consciously identifying each digit individually, the movement will more and more be also guided by the cognitively integrated template of action trajectories. So at later stages, visually identifying the individual digits becomes less relevant. The first movement (e.g., toward "2") may still be explicitly perceived, but the transition to the next key is increasingly predicted and executed based on the previously established mental structure. This means that, while the numerical code remains abstractly embedded within the action pattern, the task can now be successfully performed without the need for conscious processing of each individual digit.



Images: When unlocking a code-based lock, the numerical combination is initially followed sequentially and explicitly, with each key being individually identified and pressed. However, after the first execution, a cognitive representation of the action trajectory shapes between the keys starts to emerge, progressively optimizing the execution. The attached illustrations represent only a selection of possible abstractions of action trajectory shape patterns that emerge through repeated task execution. A crucial aspect in all templates is that the first action trajectory shape—guiding the index finger's path to the initial key—must always be executed. While this trajectory is only explicitly represented in figure 1, it remains a fundamental component of the overall action structure, regardless of the degree of automation.

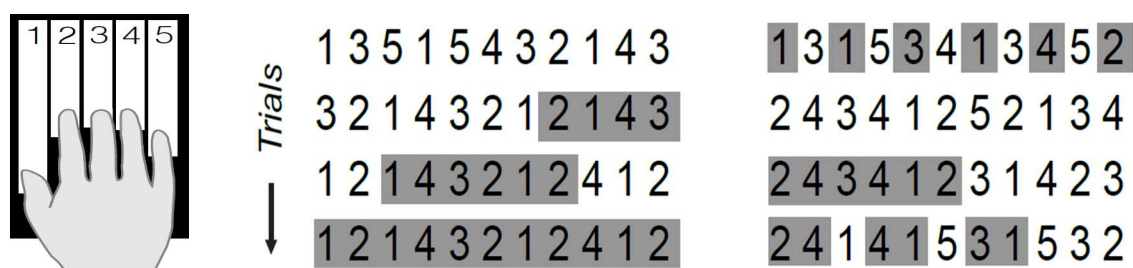
One might now consider the numerous possible ways in which the lock could be opened. However, this is not the core of the analysis. More important than identifying the number of potential pattern variations is recognizing that during the execution of familiar keypad combinations, a hybrid form of perceptual processes almost always emerges. These processes do not need to be continuously present in full but must instead dynamically supplement one another. This results in an ecologically optimized interaction process in which visual, haptic, and cognitive representations are continuously integrated to execute the task as efficiently as possible.

In situations where visual information is entirely absent, such as in complete darkness, unlocking the lock primarily relies on haptic perception and the cognitively integrated action template. However, under normal circumstances—such as in daylight—perceptual processes dynamically interweave. The user simultaneously relies on both the action trajectory shape pattern and the symbolic representation of the numerical code, with the balance between these processes being continuously adjusted. This confirms that motor execution is not a static "automation" process but rather consists of two cognitively regulated mechanisms which continuously cooperate to determine the most efficient execution strategy.

c. Templates of action trajectory shapes in sequential actions – From 3D to 2D representations

Within this paragraph the autonomous shape of one single action trajectory was first examined. Subsequently, the task of entering a numerical code via a keypad revealed a clear three-dimensional (3D) action trajectory pattern. Recent scientific studies employ experimental setups in which action trajectory shape representations emerge in an identical manner after repeated task execution.

In these experiments, however, the patterns are less directly observable because the movements are constrained to a two-dimensional (2D) plane rather than unfolding in a three-dimensional space. This constraint makes recognizing the action structure more challenging, even though the cognitive principles underlying these representations remain identical.

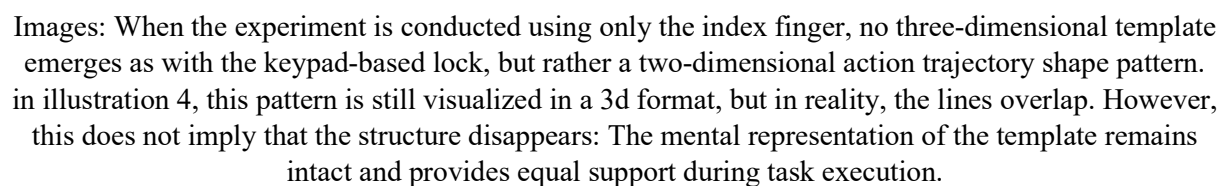


Images: The scientific research highlighted in this article involves a keypad task requiring the sequential entry of a numerical code. In the original experiment, an 11-digit sequence is entered, with the task performed using all five fingers of one hand, where each finger is assigned to a fixed key.

This section specifically examines the study: "*Repetition Effects Reveal the Sub-Sequence Representation of Actions.*"³ To clarify the core principles of this experiment, the explanation is divided into two parts. First, the study is discussed in the context of using only a single finger, making the underlying action trajectory structures more apparent. Subsequently, the original experiment is analyzed, in which the task is performed using all five fingers of one hand, with each finger assigned to a fixed key. This approach makes it evident that all discussed action trajectory shape representations are based on identical cognitive principles.

The keypad task with a single finger (modification)

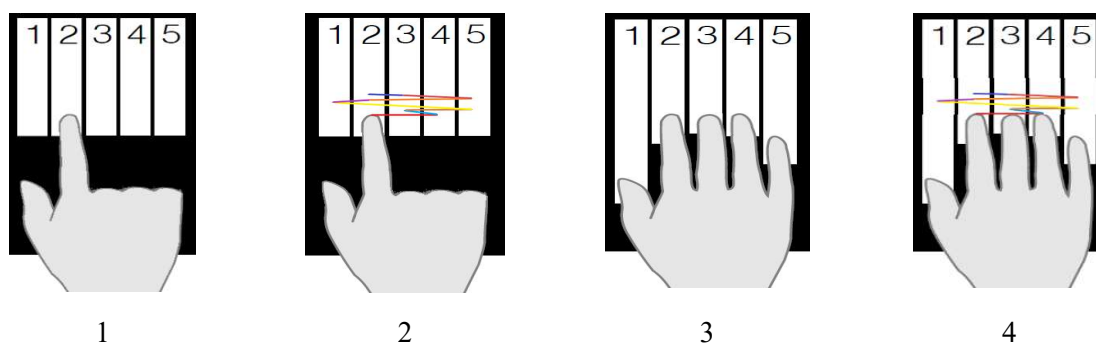
³ Shahbazi, M., Pruszyński, J. A., & Diedrichsen, J. (2024). Repetition effects reveal the sub-sequence representation of actions. *bioRxiv*. <https://doi.org/10.1101/2024.08.07.607016>



As with the numerical lock in the previous section, one could analyze all possible execution strategies here, but that is not the essential point. More importantly, the findings indicate that for a highly familiar 11-digit sequence, a hybrid perception strategy is almost always applied. This means that the user simultaneously draws on both the numerical code and the cognitive template of action trajectory shapes.

It is unlikely that someone will continue entering a frequently used numerical code purely based on the numerical representation, as they would during the first attempt. Only in complete darkness, where visual feedback is entirely absent, can the task be performed solely based on haptic perception and the automated template of action trajectory shapes. Under normal circumstances—such as in daylight—overlaps between these perception modalities occur, leading to an ecologically optimized interaction that supports the most efficient execution of the task.

In the original scientific study *Repetition Effects Reveal the Sub-Sequence Representation of Actions*, each finger was permanently assigned to one of the five keys. This makes the action trajectory shape pattern more abstract and less directly observable. However, the following analysis demonstrates that the cognitive principles remain identical in both implementations, regardless of whether the task is performed with one finger or multiple fingers.

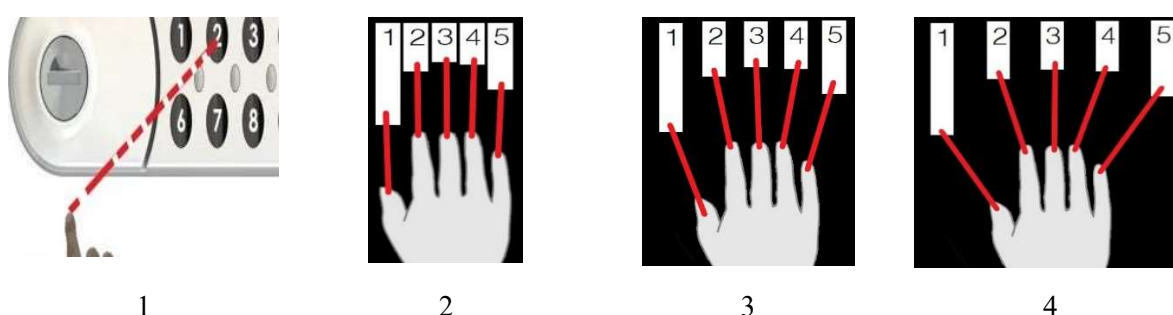


Images: Whether the numerical sequence is executed with just one finger (1) or with all five fingers (3), the resulting two-dimensional action trajectory shape template remains identical (2,4). This highlights that the cognitive structure of the action is independent of its physical execution and sensory input.

This finding underscores that the cognitive template remains consistent, regardless of the applied motor strategy. It confirms that action trajectory shapes are universal cognitive representations, independent of specific motor execution but fundamentally structured by the underlying sequential organization.

d. Expansion of the scientific research: How fingers reach keys c.q. the explanation of piano playing

In the previous subsection, both an adaptation of the original experimental setup and the initial configuration of the research study were examined. This final section proposes an extension of this specific scientific research to investigate how participants move their fingertips toward the keys before commencing the assigned task. The explanation of this phenomenon serves as the essential connecting link that unifies all components of this section, demonstrating that knowledge of templates of action trajectory shapes exhibits significant cognitive overlaps. As a result, it becomes entirely evident that inserting a car key into an ignition slot and pressing keys on a numerical lock fundamentally follow the same perceptual and executional processes as touching piano keys. This confirms that the cognitive representation of action trajectory shapes within a template is universally applicable across diverse motor tasks.



Images: Moving five fingers toward five different keys is essentially the formation of five autonomous action trajectory shapes (red) from a single fingertip. However, the task does not increase in complexity in a linear fashion. Within the template of five action trajectory shapes, the individual lines also provide a reference frame for each other. As a result, the overall template becomes more significant than the individual lines⁴.

⁴ This also applies to activities such as juggling, which will be examined in the following section.

Just like in a piano, the keys in the scientific experiment are ergonomically positioned so that most of the population, assuming normal finger length and width, does not require extraordinary actions to reach them. This design not only minimizes the likelihood of errors but also suggests that the action trajectory shapes that emerge when touching the keys form an optimized cognitive structure.

The size and width of the keys in this experiment (image 2) strongly resemble piano keys. However, during the first execution of the task—just like inserting a car key into an ignition slot or operating a numerical lock—visual control is likely required to verify whether the fingers are correctly positioned. Yet, after just one execution, one complete template image of five action trajectory shapes will implicitly emerge through execution, reducing the necessity for visual control until it ultimately disappears.

This principle becomes immediately apparent when the keys are spaced further apart (images 3 and 4). Initially, visual perception will also be required in here to ensure successful execution. However, after a short period, the complete action trajectory shape pattern formed by the execution of the separate finger movements enables entirely successful execution without any visual input.



This provides a compelling explanation for the extraordinary proficiency that musicians can achieve in piano playing. Such expertise can only be attained because the perception of individual keys is gradually replaced by a mental representation of the entire action pattern. Instead of visually identifying each note separately, a cognitive action template emerges that encompasses both the sequence and the movements between the keys. This demonstrates that skills such as piano playing and numerical lock operation are not built upon isolated movements but rather on integrated action patterns that structure and optimize motor execution at a cognitive level.

3. Additional examples: Support and general applicability

This paragraph presents additional examples with a twofold purpose: 1. They serve as reinforcement and validation of the explanation presented in the discussed scientific research, and 2. They enhance the understanding that this phenomenon is not limited to specific tasks but represents a universal characteristic of motor actions, where templates of action trajectory shapes form the foundation for efficient execution.

The shape of a single action trajectory or the template of multiple action trajectory shapes is a fundamental and universal aspect of every motor execution. However, in most actions, this structure remains largely implicit, making it difficult to explicitly determine its role. The examples discussed in this section are distinct in that their connection to the previously introduced model is not only significant but also provides insight into how templates of action trajectory shapes function as fundamental cognitive structures in motor execution.

a. Additional example I – The knight's tour puzzle

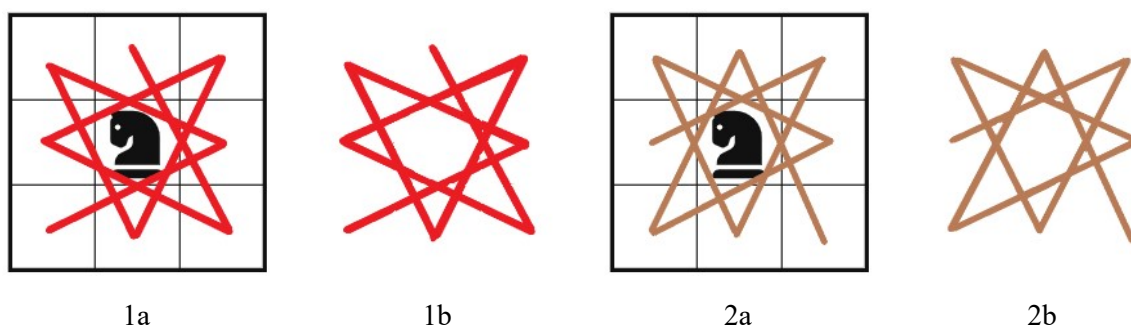
The knight's tour puzzle⁵ is a well-known phenomenon within the puzzle genre and provides a unique insight into the interaction between cognitive systems. This example is particularly relevant because it demonstrates the exact opposite mechanism of the process previously discussed in the context of keypad lock operation.

When entering a code into a keypad lock, the numerical sequence is initially perceived and entered consciously, after which the template of action trajectory shapes gradually forms, allowing to take over the motor execution. This process relies on the progressive automation of a cognitively stored sequence of actions.



In contrast, in the knight's tour puzzle⁶, the template of action trajectory shapes is already known, but the correct numerical or letter-based sequence still needs to be actively discovered. The movement sequence is, to some extent, pre-programmed by the rules of the puzzle. This means that the spatio-temporal template of action trajectory shapes is predetermined, while the symbolic-numerical information is inferred afterward.

This contrast highlights that both cognitive systems—the numerical/symbolic representation and the action trajectory shape representation—can function autonomously but also interact dynamically depending on task demands. In one case, symbolic knowledge (the numerical code) facilitates the emergence of a template of action trajectory shapes, while in the other, the action trajectory shape representation acts as a cognitive anchor to derive the correct numerical or letter-based sequence.



Images: The animations 1b and 2b are just two examples of templates of action trajectory shapes generated from the possible moves in a knight's tour. While the number of possible sequences is finite, it remains sufficiently large to allow for a degree of problem-solving ("puzzling").

This example confirms that action trajectory shapes and numerical representations can exist independently but also function complementarily. It demonstrates that cognitive representations can operate in both a 'top-down' manner (from abstract knowledge to motor execution) and a 'bottom-up'

⁵ A Knight's Tour puzzle is a word puzzle in which a hidden word must be discovered using a given 3×3 grid containing nine squares. Each square on the outer edge contains a letter. To find the word, you must move in an L-shaped pattern, similar to the movement of a knight in chess. Starting from an unknown letter, you skip two adjacent letters, selecting the third one as the next letter in the word. By continuing this pattern around the grid, a word will eventually form. The challenge in these puzzles lies in the fact that the starting letter is unknown, as is the direction of movement. The sequence can proceed clockwise or counterclockwise, adding to the difficulty.

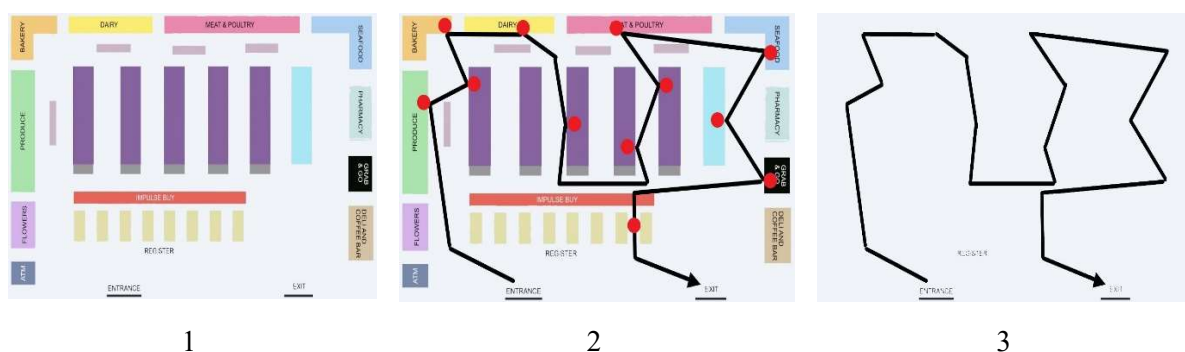
⁶ Solutions: ILLUSION and PORTUGAL.

manner (from motor execution to abstract knowledge). Moreover, it situates previous analyses of keypad lock operation within a broader cognitive framework, where the interaction between different forms of knowledge plays a crucial role.

b. Additional example II – Store navigation

Navigating through a store (e.g., a supermarket) provides a clear illustration of how templates of action trajectory shapes develop and optimize through repeated task execution, particularly involving full-body movement.

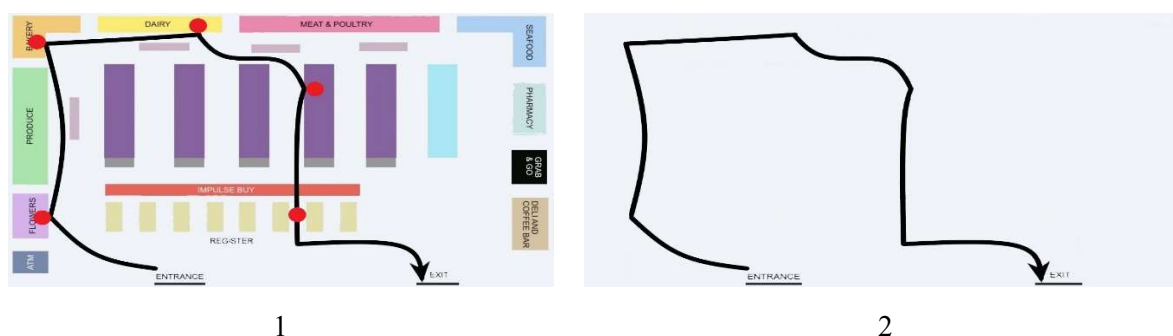
When a person visits a new store for the first time, their route is initially determined by the conscious identification of individual products on a shopping list. During this first visit, the focus remains on explicit symbolic recognition and localization of products, resulting in a chaotic and inefficient shopping route. At this stage, the user has not yet developed a cognitive representation of the most optimal organization of individual action trajectory shapes.



Images: When visiting a new store (1), the user initially navigates based solely on the items from the shopping list. In this phase, the task relies heavily on symbolic recall, similar to entering a numerical code on a keypad. However, with repeated visits, these movements progressively shape a template of action trajectory shapes, leading to a more effective and efficient shopping route. As in previous examples, it is highly probable that users will complete shopping tasks using a hybrid form of these two cognitive processes. They will alternate between recognizing the specific locations of individual groceries (2) and relying on the overall perceptual image of the entire template (3) of action trajectory shapes. In this latter case, perception is particularly linked to the *tau*-value (time-to-contact), which plays a crucial role in anticipating upcoming movements within the template.

Through repeated store visits, a standardized pattern of action trajectory shapes develops implicitly, functioning in parallel with symbolic recognition of the shopping list. Consequently, the execution of this task will likely adopt a hybrid cognitive approach, where both systems dynamically interact to achieve an optimal execution strategy. This represents an alternative cognitive representation of the same information, generated automatically through repeated execution.

This process strongly mirrors the interaction between numerical code recognition and templates of action trajectory shapes when using a keypad lock. Similar to unlocking a door, the original symbolic representation remains available but gradually intertwines with the spatial structure of the template. Over time, a mental representation of the most efficient shopping route emerges, in which symbolic recognition (the shopping list) and the template of action trajectory shapes (the route) integrate into a unified cognitive system. While one representation is primarily symbolic and numerical (numerical code representation) and the other is spatiotemporal and action-oriented (action trajectory shape template), both belong to the same cognitive domain and function as autonomous mental structures that guide task execution.



Images: This is an example of a template of action trajectory shapes where four fixed grocery items (for grandma) are collected. If an additional incidental item is added to the list, the figure largely remains intact. However, it becomes evident that with each additional item, the original structure becomes progressively more blurred. A crucial factor is whether the new item disrupts the existing template or if it is needed earlier or later in the sequence, significantly influencing the overall efficiency of the route.

The representation of a shopping route is not static but adapts flexibly to variable circumstances. This provides a direct answer to scientific inquiries regarding the integration of fixed and variable components within sequential structures. If a template contains four fixed products, adding a single extra item minimally alters the original structure. However, if three additional items are introduced at non-sequential positions, the structure becomes increasingly disorganized.

Within the template of a shopping route, the *tau*-value (time-to-contact for reaching the next grocery item) plays a fundamental role in cognitive anticipation of sequential movements. This mechanism enables users to dynamically optimize the timing and order of actions, ensuring that execution becomes ecologically efficient. While this process is universally present in motor tasks, it manifests particularly explicitly within store navigation, as spatial constraints and fixed movement paths imposed by the store layout shape the resulting action trajectory template.

This example demonstrates that the optimization of motor sequences via action trajectory templates is not limited to isolated tasks but represents a universal cognitive mechanism applicable to broader contexts, such as spatial navigation. Just like playing the piano or operating a numerical lock, a shopping route evolves from conscious symbolic recognition into an integrated cognitive structure, where perception, motor execution, and anticipation converge into an efficient action template.

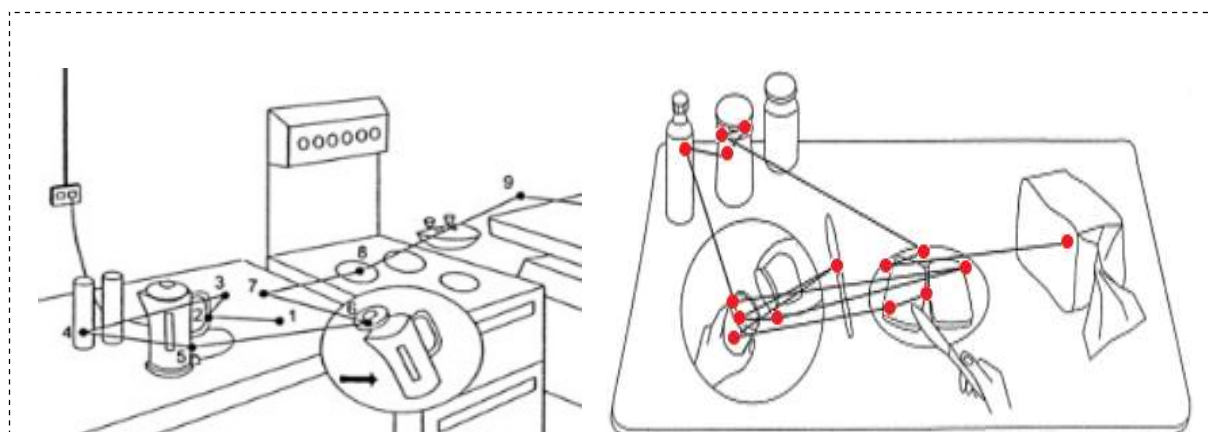
c. Additional example III – Tea making and preparing breakfast

The process of making tea and preparing breakfast provides a clear demonstration of how templates of action trajectory shapes arise implicitly and optimize through repeated task execution. This example is directly relevant to cognitive structures previously identified within numerical keypad lock operation and shopping route navigation and is based on the foundational research of Hayhoe and Land⁷, which examined eye movements and motor coordination in everyday activities. Like the previously discussed cases, this example offers empirical insights into how motor tasks transition from conscious control to structured cognitive templates.

In an unfamiliar kitchen, an individual follows a concrete tea-making script (analogous to a numerical code), primarily focusing on locating all required items. A person preparing tea in an unfamiliar environment relies entirely on visual perception to identify necessary objects (e.g., a tea bag, kettle, cups, sugar). In this phase: 1. Each script item is individually located, 2. The execution consists of serial and

⁷ Land, M. F., & Hayhoe, M. (2001). *In what ways do eye movements contribute to everyday activities?* Vision Research, 41(25-26), 3559-3565.

fragmented actions, with each sub-process actively monitored, and 3. The task is time-consuming and inefficient, as continuous visual fixation is required to guide interactions with objects.



The original animations from Hayhoe and Land's study, implicitly contained the structure of a template of action trajectory shapes, although this was never explicitly acknowledged by the researchers. Their study primarily focused on registering and analyzing visual fixation points, using eye-tracking technology to determine where and when participants visually identified specific objects within a task. In the provided images, these fixation points are shown as numbered black dots in the left figure and highlighted (added) red dots in the right figure. So the resulting template was merely a byproduct of indicating the sequential order of fixations within their research.

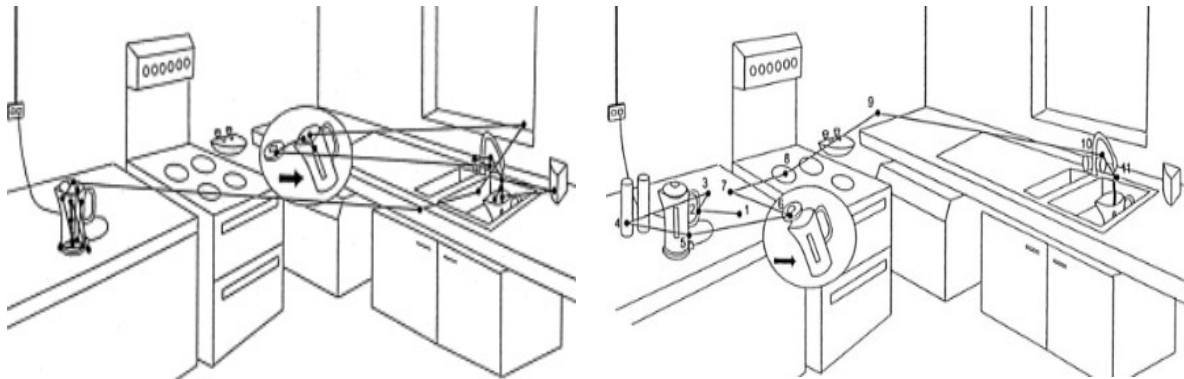
Notably, the researchers focused exclusively on fixation points rather than the movements or spaces between script items. While transitions between fixation points were recorded and visually represented in their data, they were not given substantive analysis. The underlying assumption appeared to be that these movements were simply mechanical consequences of gaze shifts and motor execution, without any intrinsic cognitive organization.

This interpretation resulted in a missed scientific opportunity, as the seemingly irrelevant "void" between fixation points actually holds the core of the template of action trajectory shapes. The movements between these points are neither random nor merely functional transitions but instead form a mentally organized sequence that reflects a cognitive pattern. In reality, this "void" reveals the underlying cognitive architecture of motor planning and task optimization while also answering the fundamental question of the "how" in sequential learning—defining the precise trajectory that must be traversed *between* (!) script items.

By analyzing only the fixation points without considering the action trajectory shape that connects them, a crucial aspect of the cognitive structure of task execution was overlooked. This research could not only have provided insights into the mechanisms of visual attention but also offered a much broader explanation of how motor and cognitive processes collaborate in the execution of complex actions. Integrating action trajectory shape structures into the analysis could have laid the foundation for a fundamental scientific breakthrough in cognitive neuroscience.

After repeated execution, a cognitive template of action trajectory shapes emerges implicitly, allowing a person to prepare tea in complete darkness within a familiar kitchen in a reasonable time frame. In an unfamiliar kitchen, however, this would be nearly impossible due to the absence of an abstract guiding template. This suggests that once sufficiently internalized, a mental representation of the template of action trajectory shapes can entirely replace visual perception.

However, in everyday practice a hybrid form of execution will emerge under normal circumstances with daylight. Similar to shop navigation. Then the task is most efficiently (ecologically) performed through a combination of both autonomous phenomena. Within which it is also important to note that the perception of script items and the action trajectory shape template will become manifest in different constellations each time a person prepares tea.



Images: If instead of tea, someone wishes to prepare coffee or hot chocolate, large parts of the tea-making script will remain the same, allowing new task-specific variations to be easily integrated into the existing pattern of action trajectory shapes. Similar to shopping navigation, this demonstrates that each separate action trajectory shape contains a perceptual representation of an autonomous *tau*-value (time-to-contact). This allows individuals to anticipate with remarkable accuracy how long the execution of a movement will take.

The cognitive representation of a template of action trajectory shapes is not static but adapts flexibly to new task conditions. If an action is incidentally added or omitted (e.g., adding sugar or not), subtle adjustments occur within the template. Larger changes, such as switching from tea to coffee preparation, retain most of the existing structure, allowing for seamless integration within the tea-making template. As seen in shopping routes and numerical lock operation, this example illustrates that action trajectory shapes are both stable and flexible, with the ability to adapt without losing their fundamental structure.

This example demonstrates that optimizing motor sequences through action trajectory shapes is not limited to isolated tasks but represents a universal cognitive mechanism applicable to broader contexts such as kitchen activities, music performance, sports, and daily motor skills. Just like playing the piano, using a numerical lock, and navigating a store, the tea-making pattern evolves from conscious visual control to an integrated cognitive structure, where perception, motor execution, and anticipation converge within an efficient action trajectory template.

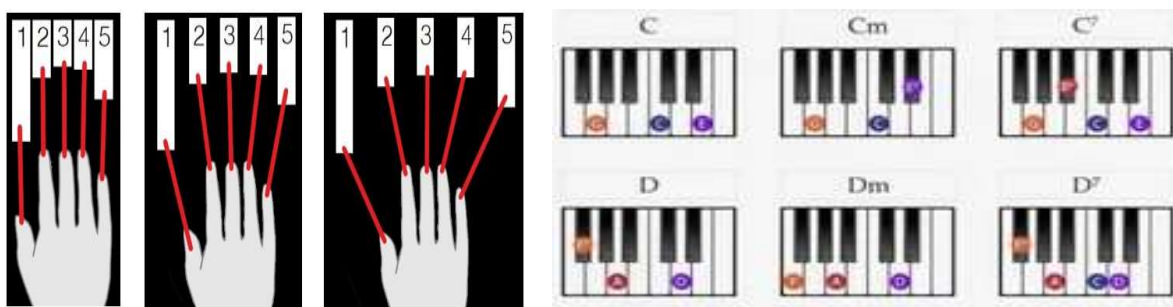
d. Additional expamle IV – Playing the piano

The process of playing the piano provides a highly suitable model for studying action trajectory shape templates within cognitive representations. When a concert pianist learns a musical piece, the initial phase of the learning process is largely based on the numeric-symbolic processing of the music score. The visual identification of notes and the corresponding determination of subjective finger placements (fingering) serve as the foundation for motor execution in this phase.

However, after repeated practice, the pianist's cognitive organization of the task changes. While the symbolic information (the score) remains available, a parallel mental representation of action trajectory shapes develops, which structures the execution of the piece. This explains why experienced pianists are capable of performing compositions largely based on an internal cognitive model, without a continuous reliance on the visual score.

This cognitive transition reflects a fundamental principle in motor sequence learning: the gradual shift from explicit symbolic recognition to an alternative action trajectory shape pattern that governs motor execution. The pianist may still be able to name the individual notes etc. of the score and be aware of

the musical structure, but the actual performance is also increasingly controlled by the internalized template of action trajectory shapes.



Images: The study of fingering reveals a strong similarity to the scientific research discussed in paragraph 2.4.. The resulting action trajectory shape template, which emerges even when playing just a few consecutive chords, becomes so complex that it is impossible to represent in a simple drawing.

In paragraph 2, it was demonstrated that a numerical combination within a keypad lock opening task can lead to an action trajectory shape pattern that autonomously contributes to task execution. The same principle applies to reading and playing from a musical score. The notation itself serves as the numerical code, where each note corresponds to specific keys. However, the template—specifically the action trajectory shape of the fingertips—is not explicitly described in the score. Instead, it emerges in precisely the same manner as in the keypad lock example. Within both piano playing and musical notation, two autonomous phenomena arise within the action trajectory shape template, aligning with the previously discussed distinction between line and shape. The *shape* of the action trajectory pattern is determined by pitch (the melody), solely representing the vertical differences between notes. The *line* within the action trajectory shape pattern corresponds to spacing (the rhythm), solely representing the horizontal differences between notes. This distinction is clearly observable in even a small section of a musical score.

Just as in the execution of the keypad lock task (see section 2), playing the piano leads to the development of an action trajectory shape template that functions alongside the explicit symbolic representation of the task. While a numerical code serves as the primary reference in the lock task, the musical score fulfills this role in piano playing. In both cases, repeated execution results in a mental representation of a pattern of action trajectory shapes, which subsequently structures task execution.

Although a musical score provides a symbolic representation of the note sequence, it does not explicitly dictate motor trajectories. Nevertheless, these movement patterns emerge spontaneously through repetition, just as an action trajectory shape pattern between the keys emerges in the keypad lock opening task. This demonstrates that motor sequences are not merely composed of discrete movements but become cognitively integrated over time.

*Edited, revised and fingered by
Richard Epstein*

Sonata IX

Abbreviations: P.T., Principal Theme; S.T., Secondary Theme; M.T., Middle Theme.

Abreviaciones: T.P., Tema Principal; T.S., Tema Segundo; T.M., Tema Medio.



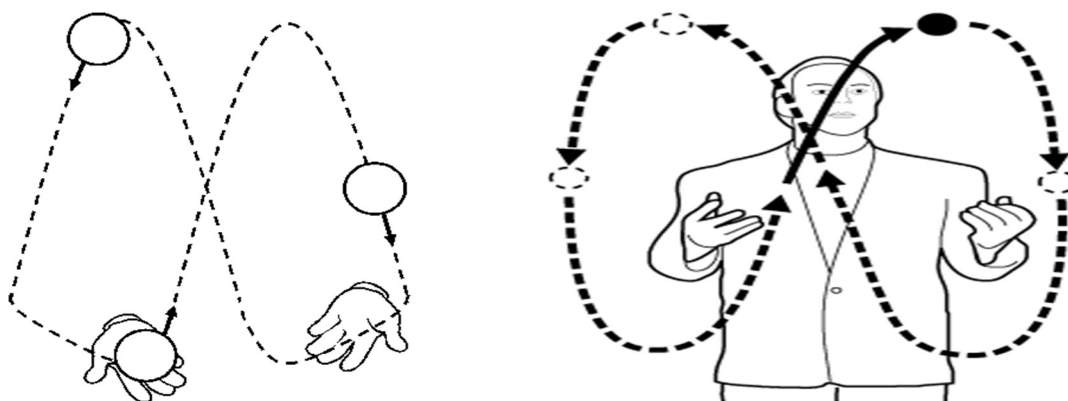
Within the pianistic action trajectory shape template, two autonomous dimensions can be distinguished, both of which can be directly identified in the musical score:

1. **The shape** (spatial organization of the template), where pitch c.q. the melody (autonomous from the rhythm) corresponds to the vertical structure of the score and is linked to the spatial displacement of the fingers across the keyboard.
2. **The line** (temporal organization of the template), where rhythm (autonomous from the melody) corresponds to the horizontal structure of the score, governing the sequential order of notes in time, thereby determining the rhythmic organization and timing.

These factually components function autonomously but interact within an integrated action trajectory shape template. This explains why experienced pianists can perform compositions largely based on mental representations, without explicit reliance on visual notation. Thus, playing the piano is a cognitively structured skill, in which numeric-symbolic knowledge and templates of action trajectory shapes often interact within a continuous dynamic hybrid memory model.

e. Additional example V – Juggling

Juggling is not considered a standard motor action but serves as a striking example of how patterns of action trajectory shapes structure complex movement patterns. The exceptional ability of jugglers to manipulate multiple objects simultaneously—such as throwing 12 rings or exchanging 9 clubs with a partner in perfect synchronization—suggests that they do not perceive and control each object individually. The explanatory model of motor action aligns with this, proposing that the mental representation of an action trajectory shape template plays a crucial role in execution.



In a basic three-ball cascade, which forms a foundation of juggling, beginners often struggle because they attempt to track each ball separately. The stabilization of the skill only seems to emerge when perception shifts away from focusing on individual objects toward an integrated action trajectory shape template. This suggests that jugglers optimize their performance by utilizing a cognitively organized structure, where the spatial and temporal coordination of all movements is integrated within a mental action trajectory pattern.

4. Scientific argument

For years, the explanatory model of the motoric movement action has focused on conveying the concept of the action trajectory shape as a fundamental cognitive phenomenon. Despite the evident role of this trajectory shape in motor execution, scientific recognition of a single action trajectory shape has proven challenging.

This situation shifted when it was demonstrated that within complex sequential actions—such as entering a code on a keypad lock—multiple action trajectory shapes self-organize into a recognizable

pattern, one that is scientifically undeniable at first glance. The recognition of multiple action trajectory shapes within a template, however, inevitably suggests that each individual trajectory shape within that template must also possess a fundamental cognitive basis. The core question in this argument is: Why would a template of action trajectory shapes be recognized as a cognitive representation, while a single action trajectory shape is not?

Every motor task, no matter how complex, ultimately consists of a series of interconnected individual action trajectory shapes. Whether it involves pressing buttons on a keypad, executing a piano passage, or carrying out a complete sequential action like making tea, each template is composed of distinct, individual action trajectory shapes. Accepting the existence of a structured pattern of trajectory shapes logically necessitates the recognition of the fundamental role of individual trajectory shapes within that pattern.

All motor actions are inherently linked within a continuous stream of movement. Action does not cease after a single movement—after grasping a coffee cup, the next logical step is drinking, followed by placing the cup back down. The segmentation of individual motor actions is conceptually arbitrary, as empirical evidence shows that movement sequences are inherently interconnected. It is, therefore, scientifically inconsistent to recognize a structured pattern of action trajectory shapes, while simultaneously denying the cognitive legitimacy of a single trajectory shape.

This breakthrough in understanding action trajectory shapes as cognitive structures within a template paves the way for a broader recognition of the universal role of action trajectory shapes in all goal-directed motor actions. It represents not only a pivotal step in the acceptance of the explanatory model of the motoric movement action, but also holds far-reaching implications for neuroscience, motor learning, and cognitive psychology.

With these insights, it becomes evident that recognizing a single action trajectory shape is both inevitable and necessary for a comprehensive understanding of motor representation and the cognitive organization of movement. Such a fundamental recognition would not only validate the explanatory model, but also mark a paradigm shift in the way we conceptualize motor control, skill acquisition, and action planning.

References

1. **Krakauer, J. W., Hadjiosif, A. M., Xu, J., Wong, A. L., & Haith, A. M.** (2019). Motor learning. *Comprehensive Physiology*. <https://doi.org/10.1002/cphy.c180008>
2. **Ghilardi, M. F., Ghez, C., Dhawan, V., Moeller, J., Mentis, M., Nakamura, T., Antonini, A., & Eidelberg, D.** (2000). Patterns of regional brain activation associated with different forms of motor learning. *Brain Research*, 871(1), 127–145. [https://doi.org/10.1016/S0006-8993\(00\)00318-3](https://doi.org/10.1016/S0006-8993(00)00318-3)
3. **Kashefi, M., Diedrichsen, J., & Pruszynski, J. A.** (2024). Motor sequence learning involves better prediction of the next action and optimization of movement trajectories. *bioRxiv*. <https://doi.org/10.1101/2024.02.10.482104>
4. **Thoroughman, K. A., & Shadmehr, R.** (2000). Learning of action through adaptive combination of motor primitives. *Nature*, 407, 742–747. <https://doi.org/10.1038/35037575>
5. **Ans, B., Coiton, Y., Gilhodes, J. C., & Velay, J. L.** (1994). A neural network model for temporal sequence learning and motor programming. *Neural Networks*, 7(8), 1265–1277. [https://doi.org/10.1016/S0893-6080\(05\)80168-7](https://doi.org/10.1016/S0893-6080(05)80168-7)
6. **Jordan, M. I., Flash, T., & Arnon, Y.** (1994). A model of the learning of arm trajectories from spatial deviations. *Journal of Cognitive Neuroscience*, 6(4), 359–376. <https://doi.org/10.1162/jocn.1994.6.4.359>
7. **Wu, H. G., Miyamoto, Y. R., Castro, L. N. G., & Ölveczky, B. P.** (2014). Temporal structure of motor variability is dynamically regulated and predicts motor learning ability. *Nature Neuroscience*, 17(2), 312–321. <https://doi.org/10.1038/nn.3616>
8. **Dezfouli, A., & Balleine, B. W.** (2012). Habits, action sequences, and reinforcement learning. *European Journal of Neuroscience*, 35(7), 1036–1051. <https://doi.org/10.1111/j.1460-9568.2012.08050.x>
9. **Wong, A. L., & Goldsmith, J.** (2016). A motor planning stage represents the shape of upcoming movement trajectories. *Journal of Neurophysiology*. <https://doi.org/10.1152/jn.00941.2015>
10. **Conditt, M. A., & Mussa-Ivaldi, F. A.** (1999). Central representation of time during motor learning. *Proceedings of the National Academy of Sciences*, 96(20), 11625–11630. <https://doi.org/10.1073/pnas.96.20.11625>
11. **Shahbazi, M., Pruszynski, J. A., & Diedrichsen, J.** (2024). Repetition effects reveal the subsequence representation of actions. *bioRxiv*. <https://doi.org/10.1101/2024.08.07.607016>
12. **Ariani, G., Shahbazi, M., & Diedrichsen, J.** (2025). Cortical areas for planning sequences before and during movement. *Journal of Neuroscience*. <https://doi.org/10.1523/JNEUROSCI.1300-24.2025>
13. **Shahbazi, M., Pruszynski, J. A., & Diedrichsen, J.** (2024). Neural correlates of online action preparation. *Journal of Neuroscience*. <https://doi.org/10.1523/JNEUROSCI.1880-23.2024>
14. **Dahm, S. F., Sülzenbrück, S., & Rieger, M.** (2025). Improvement by imagining actions: Bimanual transfer effects after action imagery practice in a sequential reaction time task. *Acta Psychologica*. <https://doi.org/10.1016/j.actpsy.2025.104011>

15. **Dyck, S., & Klaes, C.** (2025). Faster implicit motor sequence learning of new sequences compatible in terms of movement transitions. *npj Science of Learning*.
<https://doi.org/10.1038/s41539-025-00196-x>
16. **Hardwick, R.** (2024). Effects of physical practice on the duration of motor imagery. *OSF Preprints*. <https://doi.org/10.31219/osf.io/6yq8b>
17. **Hamano, Y. H., Sugawara, S. K., & Yamamoto, T.** (2025). The left primary motor cortex and cerebellar vermis are critical hubs in bimanual sequential learning. *Experimental Brain Research*. <https://doi.org/10.1007/s00221-025-07011-4>